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Code Development for Control Design Applications

Phase I: Structural Modeling

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ABSTRACT

The design of integrated controls for a complex system like a wind turbine relies on a system model in an explicit format, e.g., state-space format. Current wind turbine codes focus on turbine simulation and not on system characterization, which is desired for controls design as well as applications like operating turbine modal analysis, optimal design, and aeroelastic stability analysis. We initiated development of a specialized code to provide explicit system models. The code draws heavily from modern multibody modeling concepts as well as advanced features of an existing helicopter code. The code will be implemented in two phases: structural modeling followed by aerodynamic modeling. This paper reviews structural modeling that comprises three major steps: formulation of component equations, assembly into system equations, and linearization. Linearization provides system equations in descriptive formats, clearly delineating linear and nonlinear parts, which can then be readily used by optimal control schemes.

INTRODUCTION

A wind turbine is a complex machine that operates under severe dynamic and aerodynamic conditions. A multi-input multi-output control system offers the potential to coordinate machine component functions and improve performance, fatigue life, and stability. Central to integrated control efforts is the availability of wind turbine explicit models. Examples of explicit models are state-space models, finite element models, and modal models. An explicit model also separates linear and nonlinear parts of system governing equations in forms that can be readily integrated into systematic control design schemes. Currently available wind turbine codes, e.g. ADAMS[1], FAST[2], and YawDyn[3], have been successfully used to model a broad range of wind turbines. However, these codes rely on implicit formulation that is adequate for simulation but not for designing controllers, computing operating modes, and other important applications. ADAMS has comprehensive modeling capabilities and can generate a first-order explicit state-space model. However, a state-space model can be extracted only for a non-operating (parked) wind turbine. The accuracy of the model deteriorates rapidly as the rotor speed increases. Another limitation is that the extracted model offers only numerical information and not symbolic information, in terms of system parameters and degrees of freedom, that helps cover a wider range of design and operating conditions. System identification techniques may be used to extract low-order models[4] from simulation codes. These techniques, however, provide only single-operation-specific numerical information and require an inordinate amount of time and system identification expertise. Also, the fidelity of such models is limited only to the first few system modes. For a complex system like a wind turbine, such identified models may not capture system cross-couplings, e.g., yaw-flap-pitch, with the accuracy required for multi-input multi-output controls design. Models extracted via system identification therefore may at best be used for simple applications, e.g., power regulation or flap loads alleviation alone.

We initiated development of a specialized code to complement existing turbine codes and provide explicit system descriptive models. The code draws from newly developed flexible multibody modeling techniques[5,6] and selective features of an advanced helicopter code called UMARC[7], that is well validated and extensively used in the helicopter field. Adaptation of sophisticated features from the helicopter code such as a finite element technique specialized to rotating blade, multiblade coordinate transformation, and Floquet analysis of periodic systems, would save us several years of development and validation efforts. A detailed rationale behind this approach for code development and associated modeling issues were presented[8] at the 1998 Windpower conference held in Bakersfield, California. The code will be developed in two phases: phase I will cover structural modeling and phase II will cover full aeroelastic modeling.

This paper reviews our effort under way on structural modeling. We first describe wind turbine structural idealization, which forms the basis for all subsequent derivations. We then derive turbine component equations of motion. Next, the component equations are assembled to satisfy inter-component force and displacement constraints. This is followed by linearization of the system equations and transformation into explicit formats.

WIND TURBINE IDEALIZATION

We idealize the turbine structure by replacing it with an assemblage of flexible and rigid bodies joined by actuator elements and constraints, some of which may be time-variant. Each of these bodies may undergo large rotational and translational motions. The blades are idealized as rotating flexible beams, which may be single-path (for conventional blades) or multiple-path (for blades with multiple spars and linkages that transmit loads to the hub). Each beam undergoes flap bending, lag bending, elastic twist and axial deflection, and may have arbitrary spanwise distributions of mass, section inertia, flexural stiffness, torsion stiffness, built-in twist, and offsets amongst the elastic axis, the centers-of-mass axis and the tension-centers axis. The hub, the generator, the nacelle, the gearbox inertia, and the bed frame are treated as rigid bodies. The tower and the drive-train shaft are treated as flexible beams. The turbine model has provisions for nonlinear spring dampers to restrain any joint motion, e.g. yaw, teeter and nacelle tilt. There are also provisions for arbitrary number of blades, precone, pitch control, and delta-3 effects. This results in a comprehensive turbine model that captures all the structural mechanisms and couplings required for high-fidelity loads and response analysis, stability evaluation, modal analysis, and controls applications. For detailed stress analysis at a critical location, which may for example be required for fatigue life calculations, loads and response output from the comprehensive code may be input to any commercial finite element code that models in detail only a small region enclosing that location.

SYSTEM COORDINATES

Much of the current research in multibody dynamics addresses the selection of system generalized coordinates that describe time-dependent system configuration. The selection profoundly effects the efficacy of each of the three major steps involved in system modeling, i.e., component modeling, assembly, and linearization. A trade-off must be made between the generality and the efficiency of the dynamic formulation. For example, the choice of absolute coordinates, wherein all the degrees of freedom are referred to a single inertial frame, makes the assembly process trivial; however, for a system with rotating parts, it leads to erroneous linearization. Incorrect linearization results because some important centrifugal terms, that depend on rotational speed and are linear when referred to a rotating natural frame, become nonlinear when referred to the inertial frame, and linearization drops these terms. That is why ADAMS offers excellent simulation capabilities that rely on assembled equations, but fails to provide correct system modes that rely on linearized equations. The choice of coordinates has an even more pronounced effect on the number of system equations, the simplicity of each equation, computability of constraint forces, numerical conditioning of equations, and the efficiency of the solution procedures. We made and are still making effort to study these issues as best as we can. Once we have conclusive results, a report would follow. Basically, for multi-rigid-body dynamic modeling, we have three choices for system coordinates: absolute configuration coordinates, joint variables, and generalized speeds.

The choice of absolute coordinates leads to similar-looking equations for each body and makes assembly straightforward. This choice, however, leads to a nonminimal number of system equations. For a system with n degrees of freedom and m constraints, the number of nonminimal system equations would be n+2m which comprise n+m differential equations associated with the n+m absolute generalized coordinates, and m algebraic equations associated with the m constraints. These equations are solved for the n+m absolute coordinates and m Lagrange multipliers associated with the constraint forces. The resulting mixed set of differential-algebraic equations, however, is extremely difficult to solve accurately and a special technique, like the one proposed by Wehage[9], must be employed. The extra 2m equations also make this choice computationally expensive. Coordinate partitioning may be used to eliminate the dependent coordinates; however, this can be a tricky process. The second choice, joint variables, wherein the system equations of motion are written in terms of joint degrees of freedom, leads to a minimal set of differential equations and hence substantial computational time savings. However, it requires relatively complex constraint-specific recursive formulation. This approach is still desirable since it leads to linear equations that yield a correct eigensolution for a system with rotating parts. The third choice for system coordinates is to use generalized speeds[10], defined as a linear combination of time derivatives of generalized coordinates. This also leads to a minimal set of equations since the constraints are implicitly taken care of during formulation. This choice also has the potential to yield efficient simple equations provided the generalized speeds are defined rightly to suit given constraints. The choice also is system-configuration-specific and does not permit the automatic assembly required for a general system. Also, an efficient interface of the rigid-body subassembly with the elastic-body subassembly is still an open research area.

Compared with rigid body dynamics, the selection of coordinates for flexible body dynamics presents a number of conceptual problems. Exact modeling of an elastic body requires infinite degrees of freedom. Therefore, the first problem is the definition of an acceptable model for the elastic body using a finite set of coordinates. In the Rayleigh-Ritz method, this problem is solved by assuming that the shape of the deformed body with respect to a reference frame can be approximated through a finite set of a specific class of functions. The finite element method is one type of Raleigh method in which the elastic body is discretized into a number of regions connected by nodes. The deformation of the elastic body with respect to a reference frame is expressed in terms of shape functions and nodal degrees of freedom associated with each region called an element. The efficacy of the finite element formulation depends to a large extent on the nature of the element nodal coordinates. This is still a field of extensive research and a number of methods have been proposed which can be roughly classified into three basic formulations: the floating reference frame of formulation, the incremental formulation, and the large rotation vector formulation. Shabana[11] provides an excellent discussion of these methods

We select joint variables for rigid bodies since it leads to a minimal set of system equations and also correct linearization. For the elastic bodies, we select a floating frame of reference formulation wherein a coordinate system is assigned to each deformable body. The large rotation and translation of the deformable body are defined in terms of the absolute motion of the body-attached reference frame; this absolute motion in turn is expressed recursively in terms of joint coordinates. The deformation of the body with respect to its reference frame is expressed in terms of the elements' nodal coordinates. It can be demonstrated that this choice leads to exact modeling of the rigid body inertia when there is no deformation. However, the deformation of the body is assumed to be moderate. This assumption is valid for wind turbine elastic components and allows substantial simplification of the governing equations.

COMPONENT EQUATIONS

Basic to full system modeling is the derivation of equations governing its components. From the section on wind turbine idealization it follows that any component of the wind turbine may be modeled either as a flexible beam or as a rigid body.

For the flexible beam, Hamilton's variational principle is used to derive component equations of motion. For a non-conservative system, this principle is expressed as

$$dp = \int_{t_1}^{t_2} \left(dU - dT - dW\right)dt = 0 \qquad (1)$$

where dU is the virtual variation of potential energy, dT is the virtual variation of kinetic energy, and dW is the virtual work done by external forces, e.g., aerodynamic forces, which are not derivable from a potential function. The virtual variation in the strain energy is given by

$$dU = \int_o^L \iint_A \left[\left(s_{xx} de_{xx} + s_{xh} de_{xh} + s_{xz} de_{xz}\right) + r\bar{g} \cdot d\bar{R}^{QI}\right] dh\, dz\, dx$$
$$= \int_o^L \iint_A \left[\left(\mathrm{E} e_{xx} de_{xx} + G e_{xh} de_{xh} + G e_{xz} de_{xz}\right) + r\bar{g} \cdot d\bar{R}^{QI}\right] dh\, dz\, dx \qquad (2)$$

The strain components e_{xx}, e_{xh}, e_{xz} are functions of the beam extensional deflection u, bending deflections n and w, the elastic twist f, and their spatial derivatives. The explicit expressions for these strains are derived by considering the orientation of a generic coordinate triad (x, h, z), attached to the principal axes of a cross section of the deformed blade, with respect to the (x, y, z) coordinate triad attached to the undeformed blade.

Detailed derivation and expressions for the strain components are provided in the UMARC Theory Manual[7]. The $r(x,h,z)$ is the beam local material density, $\vec{g}$ is the gravity vector, and $\vec{R}^{QI}$ is the position vector of an arbitrary point (x,h,z) on the blade with respect to the ground attached (inertial) frame. The expression for the kinetic energy T for the flexible blade is also provided in UMARC[7]. However, this expression is derived for a helicopter-specific configuration (comprising fuselage-shaft-hub sequence) and assuming moderate fuselage angular displacements. We modify the kinetic energy expression to allow for an arbitrarily large motion of the reference frame attached to the beam root. For the blade, motion of its reference frame would result from the cascaded effect of tower top motion with respect to ground, nacelle motion with respect to the tower top, drive-train motion with respect to the nacelle, the hub motion with respect to the drive train, and the blade reference frame motion with respect to the hub. The derived beam expression is general in nature and is applicable to all the flexible components, i.e., tower, shaft, and blades, with arbitrary boundary constraints. The system assembly procedure, discussed later, automatically synthesizes the cumulative effect of all the individual component motions. A detailed derivation of T is outside the scope of this paper; it will be included in a report under preparation. The final expression for the virtual variation in the kinetic energy, in a compact vector form, can be written as

$$
\begin{aligned}
dT = \sum_{i=1}^{3}\Bigg[\int_{o}^{L} du_i \Big\{ & m\Big(\ddot{\vec{R}}_{OI} + \vec{a}_{OI}\times(\vec{x}_1+\vec{u}) + \vec{w}_{OI}\times(\vec{w}_{OI}\times(\vec{x}_1+\vec{u}))\Big) \\
& + 2\vec{w}_{OI}\times\dot{u}_{i-1}\hat{h}_i + \ddot{u}_{i-1}\hat{h}_i \Big\} + \vec{a}_{OI}\times\vec{m}_h + \vec{w}_{OI}\times(\vec{w}_{OI}\times\vec{m}_h) \\
& + 2\vec{w}_{OI}\times\vec{m}_h\vec{w}_{PO} + \vec{a}_{PO}\times\vec{m}_h - 2\vec{w}_{OI}\cdot\vec{w}_{PO}\vec{m}_h \\
& + \vec{w}_{PO}\times(\vec{w}_{PO}\times\vec{m}_h) + \vec{w}^{PO}\cdot\vec{m}_h\vec{w}_{PO} - \vec{w}_{PO}\cdot\overline{w}_{OI}\vec{m}_h\Bigg]\cdot\hat{h}_i\,dx \\
+ \sum_{i=1}^{3}\Bigg[\int_{o}^{L}\Big\{ & \sum_{i=1}^{3}\frac{\partial k_i}{\partial uj''}du'_j + d_{3i}dq\Big\}\Big\{\vec{m}_h\times\Big[\ddot{\vec{R}}_{OI} + \vec{a}_{OI}\times(\vec{x}_1+\vec{u})\Big] \\
& + \vec{w}_{OI}\times(\vec{w}_{OI}\times(\vec{x}+\vec{u})) + 2\vec{w}_{OI}\times\dot{u}_{i-1}\hat{h}_i + \ddot{u}_{i-1}\hat{h}_i\Big] \\
& + r_3\vec{a}_{OI} - \vec{a}_{OI}\cdot\vec{r}_{33} + (\vec{w}_{OI}\cdot\vec{r}_{33})\times\vec{w}_{OI} \\
& + 2(\vec{w}_{OI}\cdot\overline{r}_{33})\times\vec{w}_{PO} + (\vec{w}_{PO}\cdot\overline{r}_3)\times\vec{w}_{OI} + (\vec{w}_{PO}\cdot\vec{r}_3)\times\vec{w}_{PO} \\
& + \vec{r}_3\vec{a}_{PO} - \vec{a}_{PO}\cdot\vec{r}_{33}\Big\}\Bigg]\cdot\hat{x}_i\,dx
\end{aligned}
\tag{3}
$$

where

$$\underline{\hat{x}} = \begin{bmatrix}\hat{x}_1 & \hat{x}_2 & \hat{x}_3\end{bmatrix}^T$$

and $$\underline{\hat{h}} = \begin{bmatrix}\hat{h}_1 & \hat{h}_2 & \hat{h}_3\end{bmatrix}^T = \begin{bmatrix}x & h & z\end{bmatrix}^T$$

are the unit vectors associated with the coordinate systems (x,y,z) and (x,h,z) respectively. The section integrals are defined as

$$
\begin{aligned}
& m = \iint_A r\,dxdh \;;\; \vec{m}_h = \iint_A r\hat{h}\,dxdh \\
& r_3 = \iint_A r\hat{h}\cdot\hat{h}\,dxdh \;;\; \overline{r}_{33} = \iint_A r\hat{h}\hat{h}\,dxdh
\end{aligned}
\tag{4}
$$

$\vec{x} = \begin{bmatrix}x_1 & x_2 & x_3\end{bmatrix}^T$

$\vec{u} =$ elastic axis deflection vector

$= \begin{bmatrix}u_1 & u_2 & u_3\end{bmatrix}^T = \begin{bmatrix}u & v & w\end{bmatrix}^T$

$\vec{R}_{OI} =$ position vector of the base-frame origin with respect to the inertial frame. $\vec{k} = \begin{bmatrix}k_1 & k_2 & k_3\end{bmatrix}^T$

The subscripts O,P,I represent respectively the origins of the base frame, the defamed-blade-attached frame, and the inertial frame. The vector $\vec{w}$ and $\vec{a}$ represent the angular velocity and the angular acceleration, respectively. The k_i is the curvature-like quality called the moment curvature.

Note that the axial degrees of freedom u in the above derivation is in fact a quasi-coordinate representing the resultant effect of elastic axial deformation and kinematic shortening due to beam flexure, i.e.

$$u = u_e - \frac{1}{2}\int_0^x (v'^2 + w'^2)\,dx \tag{5}$$

If the effects of the axial elastic elongation, u_e, are considered negligible, then the axial displacement u may simply be expressed in terms of the slopes v' and w' via equation (5). This would eliminate the axial degree of freedom. But, for a blade with multiple load paths, via flexbeams for example, one must use[14] the axial coordinate to avoid erroneous results. The use of u coordinate, however, results in severe numerical problems and incorrect linearization. This problem is solved[7] by using the axial elastic elongation u_e as the nodal coordinate instead of u. This leads to integro-partial differential equations. Though this necessitates computation of spatial integrals, to account for Coriolis effects, numerical stability is guaranteed and correct linearization is also assured via the following substitution:

$$\Delta u_e = \Delta u + \int_0^x (v'\Delta v' + w'\Delta w')\,dx \tag{6}$$

Finite Element Discretization

Hamilton's principle (1) results in partial differential equations for the continuous-domain flexible body. A specialized finite element technique[7], developed to handle integral-partial differential equations associated with rotating flexible blades, is used to spatially discretize the governing equations into a finite set of N ordinary differential equations in time, where N is the number of generalized coordinates representing the finite element modal degrees of freedom. Each flexible component (tower, shaft, or blade) is divided into a number of 15-degrees of freedom beam elements (Figure 1). The finite element assembly process ensures continuity of displacements and slopes for the two transverse bending deflections, and continuity of axial and elastic twist deflections. Using Hamilton's polynomials, the distribution of deflections over a beam element i is expressed in terms of the nodal displacement vector, $\mathbf{q}^i$, which consists of the fifteen nodal degrees of freedom shown in Figure 1. Formulation of beam element equations, followed by assembly, results in the full beam governing equations:

$$\mathbf{M}_b\ddot{\mathbf{q}} + \mathbf{c}_b\dot{\mathbf{q}} + \mathbf{K}_b\mathbf{q} + \mathbf{M}_{bF}\ddot{\mathbf{X}}_F + \mathbf{C}_{bf}\dot{X}_F + \mathbf{K}_{bF}\mathbf{X}_F = \mathbf{F}_b\left(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{x}, \dot{\mathbf{x}}, \mathbf{f}, q, t\right) \qquad (7)$$

where $\mathbf{q}$ is the vector of the full beam elastic degrees of freedom measured with respect to its undeformed base frame, and $\mathbf{x}_f$ is the vector of base frame absolute degrees of freedom. The **f** is the vector of externally applied forces, e.g., aerodynamic forces. The $\mathbf{M}_b, \mathbf{K}_b$, and $\mathbf{C}_b$ are the beam mass stiffness, and damping/gyroscopic matrices respectively. The matrices $\mathbf{M}_{bF}$, $\mathbf{C}_{bF}$, and $\mathbf{K}_{bF}$ represent inertial, gyroscopic, and stiffness couplings between the beam and the base frame motions. In case the beam represents the rotating blade, these coupling matrices would be periodic in time. Further, if the inflow is yawed or sheared, matrices $\mathbf{M}_b, \mathbf{C}_b$, and $\mathbf{K}_b$ would also be periodic. The q is the vector of pitch controls. The $\mathbf{F}_b$ is the vector of all constant and nonlinear forces on the beam.

An effort is under way to develop a mixed formulation for the beam component that allows arbitrary reference frame rigid body motion. In this formulation, a mix of displacements, curvatures, and momenta are selected as the nodal coordinates. This leads to a very simple set of beam equations and may be incorporated in future should it be confirmed that it wouldn't result in any assembly or linearization problems.

For a rigid component, the key issue is the choice of orientation coordinates. Euler angles, used by most of the earlier multibody dynamic codes to represent rigid body orientation, result in well-known singularity problems. More advanced codes use Euler parameters which, though guaranteeing avoidance of singularities, do not permit linearization[15]. Rodrigues parameters, the orientation coordinates used by the modern dynamic codes, may result in singularities, but only under highly improbable situations.[15] The overriding advantages of the Rodrigues parameters are the feasibility of linearization and simplified equations. We use the standard Newton-Euler equations for the rigid component governing equations. The angular velocities appearing in the Newton-Euler equations are developed in terms of the Rodrigues parameters and their time derivatives. These equations are used later to develop a recursive formulation for constrained multibody system. We also use Rodrigues parameters to develop a library of constraint equations for standard joints, namely, revolute joint, prismatic joint, cylindrical joint, spherical joint, sliding-cum-ball joint, screw joint, and planar joint.

ASSEMBLY INTO SYSTEM EQUATIONS

Assembly simply implies combining individual component equations into a single set of system equations by satisfying inter-component displacement and force constraints. For the flexible beam, the beam may be thought of as a super component consisting of finite element components. The Hamilton's variational principle implicitly takes care of the inter-element constraint forces since these do not contribute to any energy variation associated with configuration-compatible virtual displacements. The finite element assembly takes care of the inter-element displacement compatibility.

For rigid component inter-connections, which include interconnection of a rigid body to a flexible component via its reference frame, there are basically three assembly schemes. One is the use of Lagrange multipliers in conjunction with the usage of absolute coordinates. This greatly facilitates automated assembly of system equations because configuration of each component of the multibody system is described by a global set of generalized coordinates that are independent of the topological configuration of the system. The Lagrange multipliers take care of the inter-component constraint forces. Constraint equations are used to augment system differential equations to take care of the inter-component displacements. As pointed out earlier, this results in a mixed system of differential and algebraic equations. This augmented formulation in terms of absolute coordinates poses several problems: a) complexity of the numerical algorithms that must be used to solve the mixed systems of differential and algebraic equations; b) non-minimal number of systems governing equations; c) increased likelihood of singularities associated with orientation coordinates; and d) system linearization that may be inappropriate for modal analysis and controls applications. The second assembly approach is the Kane's approach wherein the generalized velocities are defined in terms of constrained displacement coordinates leading to a minimal set of system equations. The concept of generalized active forces implicitly takes care of the constraint forces. The third assembly scheme, the recursive formulation, is particularly suited for linearization of flexible multibody systems, wherein the governing equations are formulated in terms of the joint (relative) degrees of freedom. The constraint equations are also developed in terms of joint (instead of absolute) coordinates and are used to eliminate dependent coordinates as well as workless constraint faces. This leads to a minimal set of differential equations. The numerical procedure required for solving these differential equations is much simpler than the procedure required for solving the mixed set of equations resulting from augmented formulation. Because of these advantages, we select recursive formulation to assemble the component equations.

Recursive Formulation

Fundamental to this formulation is developing a kinematic relation between two bodies, $i-1$ and i, in terms of joint variables connecting the two bodies. For illustration we consider two rigid bodies $i-1$ and i connected by a cylindrical joint as shown in Figure 2. Most of the joints in a typical wind turbine can be idealized as a revolute joint, which as we shall see is a special case of the cylindrical joint. For other types of joints, a procedure similar to the one outlined below is followed.

The two-degree-of-freedom cylindrical joint (Figure 2) permits relative rotation f^i about, and relative translation s^i along, the joint axis $\mathbf{v}^{i-1}$. The absolute translation and rotational coordinates of body i, i.e. $\mathbf{R}^i$ and $\mathbf{t}^i$, are related to similar coordinates of body $i-1$ as follows:

$$\mathbf{R}^i + \mathbf{T}^i\mathbf{u}_p^i - \mathbf{R}^{i-1} - \mathbf{T}^{i-1}\mathbf{u}_p^{i-1} = \mathbf{v}^{i-1}\mathbf{s}^i \tag{8}$$

where u_p^i is the position vector of point $\mathbf{P}^i$ fixed in body i relative to the reference coordinate triad $\left(x^i, y^i, z^i\right)$ fixed to body i. The absolute angular velocity w^i of body i is related to the absolute angular velocity

$$w^i = w^{i-1} + w^{i-1,i} \tag{9}$$

where $w^{i-1,i}$ is the angular velocity of body i with respect to body $i-1$, and is given by

$$w^{i-1,i} = \mathbf{v}^{i-1}\dot{f}^i \tag{10}$$

The unit vector $\mathbf{v}^{i-1}$, directed along the joint axis, can be written as

$$\mathbf{v}^{i-1} = \mathbf{T}^{i-1}\bar{\mathbf{v}}^{i-1} \tag{11}$$

where $\mathbf{v}^{i-1}$ unit vector is also along the joint axis but is defined in the coordinate triad of body $i-1$, and has therefore constant components. A series of vector and algebraic operations finally yields the following equation for absolute acceleration of body i:

$$\ddot{\mathbf{Q}}^i = \mathbf{D}^i\ddot{\mathbf{Q}}^{i-1} + \mathbf{H}^i\ddot{\mathbf{Q}}_r^i + b^i \tag{12}$$

where

$$\ddot{\mathbf{Q}}^i = \left[\ddot{\mathbf{R}}^{iT} \quad a^{iT}\right]^T \tag{13}$$

$a^i = \dot{w}^i =$ angular acceleration of body i

$$\mathbf{D}^i\left(P^i, P^{i-1}\right) = \begin{bmatrix} \mathbf{I} & \tilde{\mathbf{r}}_p^{i,i-1} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{14}$$

and $\tilde{\mathbf{r}}_p^{i,i-1}$ is the skew matrix associated with the vector

$$\tilde{\mathbf{r}}_p^{i,i-1} = \mathbf{u}_p^i - \mathbf{u}_p^{i-1} - s^i\mathbf{v}^{i-1} \tag{15}$$

$$\mathbf{H}^i = \begin{bmatrix} \mathbf{v}^{i-1} & \tilde{u}_p^i\mathbf{v}^{i-1}\tilde{u}_p^i\mathbf{v}^{i-1} \\ \mathbf{0} & \mathbf{v}^i \end{bmatrix} \tag{16}$$

$\mathbf{Q}_r^i = \left[s^i \quad f^i\right]^T =$ vector of joint degrees of freedom

$$b^i = \left[\left(b_R^i + \tilde{u}_p^i b_q^i\right)^T \quad b_q^{iT}\right]^T \tag{17}$$

$$b_R^i = -w^i \times \left(w^i \times \mathbf{u}_p^i\right) + w^{i-1} \times \left(w^{i-1} \times \mathbf{u}_p^{i-1}\right) + w^{i-1} \times \left(w^{i-1} \times \mathbf{v}^{i-1}\right)s^i + 2\dot{\mathbf{v}}^{v-1}s^i$$

$$b_q^i = \left(w^{i-1} \times \mathbf{v}^{i-1}\right)\dot{f}^i \quad (18)$$

Note that the term of the matrices $\mathbf{D}^i$ and $\mathbf{H}^i$ depends on the joint type. Equations similar to (12), representing two bodies interconnected by a joint, are developed for spherical, universal, prismatic, resolute, and ball/sliding joints. The resolute joint in fact can be considered as a special case of the cylindrical joint wherein translation s^i is held constant, and the prismatic joint is also a special case of the cylindrical joint wherein the rotation f^i is held constant.

If motion of the base body i is known, equation (12) can be used recursively to express absolute acceleration of any body i in terms of the motion of the base body and the motion of all the joints that connect body i to the base body (the base body is usually the fixed ground). Use of Newton-Euler equations in conjunction with equation (12) yields a minimal set of differential equations for each body expressed in terms of the joint degrees of freedom.

In general, a multibody system, e.g., a wind turbine, consists of interconnected rigid and flexible bodies. Recursive relation (12) helps us to express the motion of reference frame of any body i in terms of the joint degrees of freedom and the motion of a base frame that may be attached to a rigid body or a flexible body, e.g., the tower top. The equation of motion for each body thus is expressible in terms of all the system joint degrees of freedom and all the system elastic degrees of freedom. These component equations are simply collected and put in the matrix form:

$$\begin{bmatrix} \mathbf{M}_{1J} & \mathbf{M}_{1E} \\ \mathbf{M}_{1J} & \mathbf{M}_{2E} \\ \vdots & \vdots \\ \mathbf{M}_{1J} & \mathbf{M}_{NE} \end{bmatrix} \begin{pmatrix} \ddot{\mathbf{q}}_J \\ \ddot{\mathbf{q}}_E \end{pmatrix} + \begin{bmatrix} \mathbf{C}_{1J} & \mathbf{C}_{1E} \\ \mathbf{C}_{1J} & \mathbf{C}_{2E} \\ \vdots & \vdots \\ \mathbf{C}_{1J} & \mathbf{C}_{NE} \end{bmatrix} \begin{pmatrix} \dot{\mathbf{q}}_J \\ \dot{\mathbf{q}}_E \end{pmatrix} + \begin{bmatrix} \mathbf{K}_{1J} & \mathbf{K}_{1E} \\ \mathbf{K}_{1J} & \mathbf{K}_{2E} \\ \vdots & \vdots \\ \mathbf{K}_{1J} & \mathbf{K}_{NE} \end{bmatrix} \begin{pmatrix} \mathbf{q}_J \\ \mathbf{q}_E \end{pmatrix} = \mathbf{F}_{NL} \quad (19)$$

where $\mathbf{q}_J$ and $\mathbf{q}_E$ represent the system joint and elastic degrees of freedom, respectively. Expressing the system degrees of freedom as

$$\mathbf{x} = \left[\mathbf{q}_J^T \quad \mathbf{q}_E^T\right]^T \quad (20)$$

equation (19) can be put in the compact form:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}_{NL} \quad (21)$$

A parallel effort is under way to extend Kane's assembly scheme to include flexible components. It appears that for the flexible components, a mixed formulation mentioned earlier would be required to make assembly feasible. Should we succeed, we would perform a comparative study and select an assembly approach that would be most advantageous in terms of automation, linearization, simplicity of equations, and computational time.

LINEARIZATION

System equations in the implicit form, that are generally used by simulation codes, can be expressed as

$$\mathbf{g}(\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}, \mathbf{f}, t) = 0 \quad (22)$$

where $\mathbf{x}$ is the vector of system coordinates, $\mathbf{f}$ is the vector of applied, e.g., aerodynamic forces, and t represents time. In the explicit form, the system equations, resulting from the assembly of flexible component equations and rigid component equations are expressed as

$$\mathbf{M}(t)\ddot{\mathbf{x}} + \mathbf{C}(t)\dot{\mathbf{x}} + \mathbf{K}(t)\mathbf{x} = \mathbf{F}_{NL}(\mathbf{x}, \dot{\mathbf{x}}, t, \mathbf{w}, q) \quad (23)$$

where $\mathbf{w}$ is the vector of wind velocity components, and q is the vector of controls, e.g., pitch angles of blades, which may be explicit functions of time and/or the system coordinates $\mathbf{x}$. $\mathbf{M}(t)$, $\mathbf{C}(t)$, and $\mathbf{K}(t)$ are in general periodic functions of time. $\mathbf{F}_{NL}$ is the vector of constant and nonlinear forces. Expressing $\mathbf{x}(t)$ as a perturbation about the periodic solution, i.e.

$$\mathbf{x}(t) = \mathbf{x}_0(t) + \Delta\mathbf{x}(t) \quad (24)$$

equations (23) become

$$\mathbf{M}(\ddot{\mathbf{x}}_0 + \Delta\ddot{\mathbf{x}}) + \mathbf{C}(\dot{\mathbf{x}}_0 + \Delta\dot{\mathbf{x}}) + \mathbf{K}(\mathbf{x}_0 + \Delta\mathbf{x}) = \mathbf{f}_0 + \left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right]_{x0} \Delta\mathbf{x} + \left[\frac{\partial \mathbf{f}}{\partial \dot{\mathbf{x}}}\right]_{\dot{x}0} \Delta\dot{\mathbf{x}} + \cdots \quad (25)$$

or

$$\mathbf{M}\Delta\ddot{\mathbf{x}} + \left(\mathbf{C} - \left[\frac{\partial \mathbf{f}}{\partial \dot{\mathbf{x}}}\right]_{\dot{x}0}\right)\Delta\dot{\mathbf{x}} + \left(\mathbf{K} - \left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right]_{x0}\right)\Delta\mathbf{x} = \mathbf{f}_{NL}(t, \mathbf{x}, \dot{\mathbf{x}}) \quad (26)$$

where $\mathbf{f}_0$ represents constant forces, e.g., time-invariant centrifugal forces, and $\mathbf{f}_{NL}$ represents all nonlinear terms. Equation (26) represents the final set of system linearized equations; the left-hand side comprises the linear part and the right-hand side comprises the nonlinear part. To compute modal frequencies and vectors, we set the right-hand side to zero and use the Floquet approach.[12,13] These modes then may be used to transform equation (26) into the modal domain. Either these modal equations or the physical-domain equations (26) can be transformed into the first-order state-space format for use in controls design.

CURRENT STATUS AND FUTURE WORK

We derived the wind turbine component equations, comprising both flexible and rigid parts. For the rigid parts, we attempted three approaches: Lagrangian formulation based on global coordinates and Lagrangian multipliers, recursive formulation using joint coordinates, and Kane's approach using partial speeds. This allows a wide choice of assembly options. A library of joint constraints, based on Rodrigues parameters, has also been developed that can be integrated into any of the assembly schemes. A scheme to symbolically generate linearized equations has also been developed. Figure 2 shows the organization of the computer code covering structural modeling. The code is being developed in a modular fashion to allow efficient data management, future modifications and expansions. Solid boxes indicate modules that have been completely coded. Boxes in dashed lines indicate modules under development.

Both the component and system would be validated with exact results, if available, and with other codes, e.g., the ADAMS code, using specific forcing functions and simulations. This will be followed by the integration of structure code with unsteady aerodynamics and dynamic induced inflow models in state-space formats. The resulting aeroelastic code will be validated with ADAMS for typical wind turbine configurations and specific simulations. Extensive results will be presented to demonstrate the ability of the code to compute operating modes, stability, and aeroelastic descriptive models in diverse formats.

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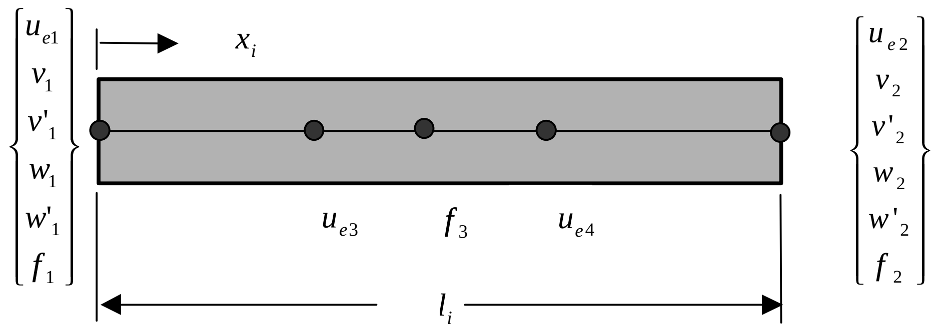

Figure 1. Fifteen degrees of freedom finite element used to discretize the flexible beam

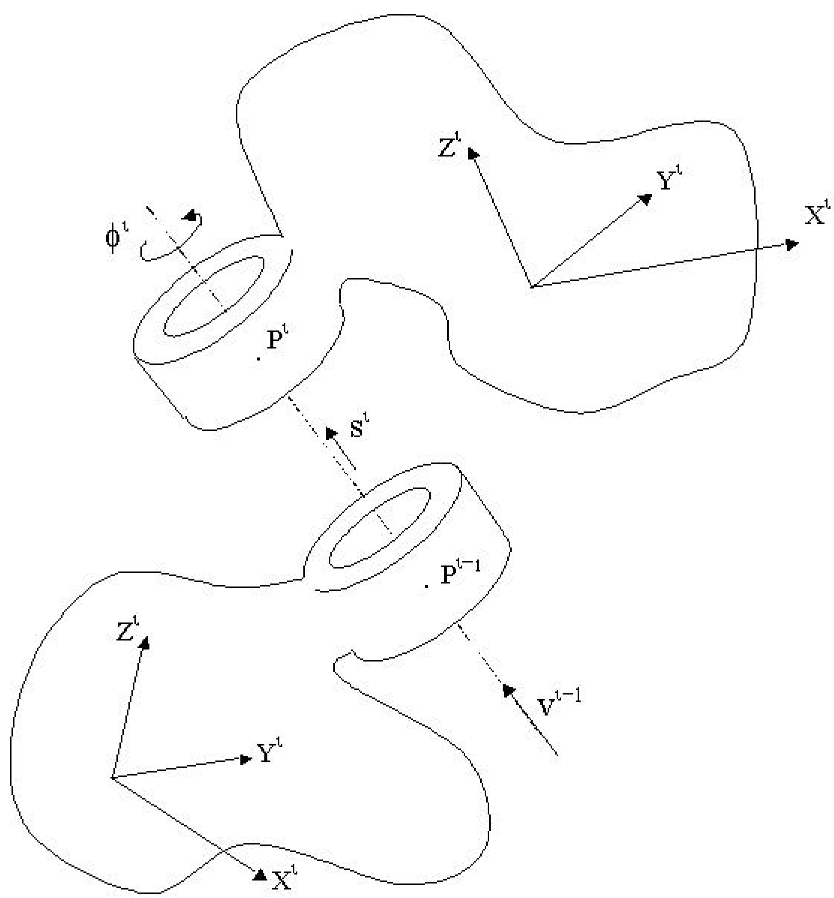

Figure 2. Relative motion of body i with respect to body $i-1$.

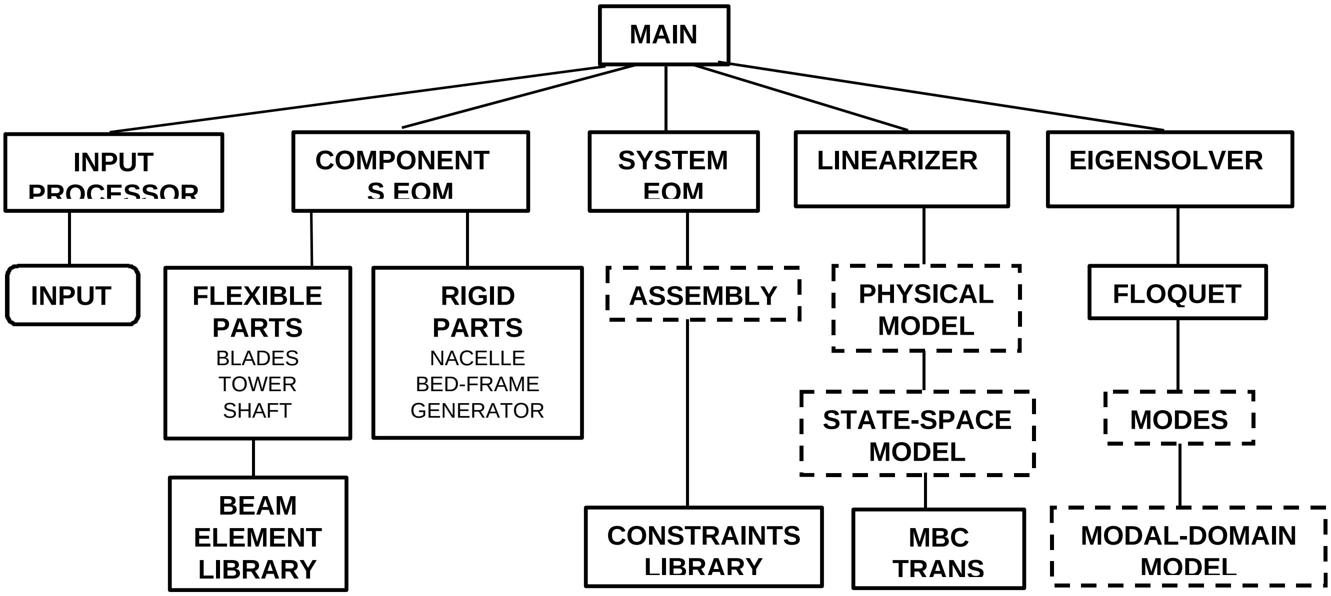

Figure 3. Major modules of the structures code (solid lines identify completed modules, dashed lines identify modules under development)